Mathematics Framework
Garden City Public Schools

Statement of Purpose:
To support mathematical proficiency and to meet the challenges of preparing students for College and Career, Garden City Public Schools has developed the following mathematics framework. It provides a synthesis of research-based principles and strategies proven effective in promoting all students’ mathematics development—including the critical, creative, and self-regulated thinking processes that underlie the Kansas College and Career Ready Standards (KCCRS).

The KCCRS calls for a shift to focus on sense-making, reasoning, and connections to real-world situations. Students will need knowledge and skills that prepare them to apply mathematics in a variety of contexts, including their future lives as responsible citizens. A transformation is required that results in a greater emphasis on the many ways that math helps us understand the world, and less on math for its own sake. There needs to be a focus on understanding and concepts, not just computation or procedures.

Developing and applying real-world situations requires new technology tools and new approaches to teaching and learning. It also requires new assessment methods. The goal of the assessments should be to inform students and teachers about the level of understanding achieved, and of the next necessary steps in instruction. Ongoing informal assessment that guides teaching and learning brings about increased learning as well as increased self-esteem for students.

Students will need the resources to prepare them for our rapidly changing world. By working on authentic tasks and real-life problem situations, students make connections related to their own learning of mathematics as well as important new connections among graphic, symbolic, and dynamic representations that are critical in order to understand mathematics effectively. They will also need to recognize that studying mathematics in high school is important for their future careers.

A commitment to teacher professional development is essential that is collaborative with time allotted for vertical discussions and alignment across grade levels and high school courses. Teachers will need long-term professional development and support, including opportunities for reflection on their practice and guidance in improving it.

To achieve the vision of reasoning and sense-making as the focus of students’ mathematical experiences, all components of the educational system – curriculum, instruction, and assessment – must work together and be designed to support students’ achieving these concepts and skills. Through a coherent and cohesive mathematics program with a strong alignment of curriculum, instruction and assessment, students will have the opportunity to be fully prepared for College and Career challenges.
Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it.
Effective Mathematics Teaching and Learning

An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically. (Principles to Action, NCTM)

<table>
<thead>
<tr>
<th>Mathematics Teaching Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Establish mathematics goals to focus learning.</strong> Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.</td>
</tr>
<tr>
<td><strong>Implement tasks that promote reasoning and problem solving.</strong> Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.</td>
</tr>
<tr>
<td><strong>Use and connect mathematical representations.</strong> Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.</td>
</tr>
<tr>
<td><strong>Facilitate meaningful mathematical discourse.</strong> Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.</td>
</tr>
<tr>
<td><strong>Pose purposeful questions.</strong> Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.</td>
</tr>
<tr>
<td><strong>Build procedural fluency from conceptual understanding.</strong> Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.</td>
</tr>
<tr>
<td><strong>Support productive struggle in learning mathematics.</strong> Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.</td>
</tr>
<tr>
<td><strong>Elicit and use evidence of student thinking.</strong> Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.</td>
</tr>
</tbody>
</table>
Standards of Mathematical Practice

The Common Core mathematical practice standards are the foundation for mathematical thinking and practice for students as well as guidance that helps teachers modify their classrooms to approach teaching in a way that develops a more advanced mathematical understanding. Think of these standards as a guide to creating a more complex and absorbing learning experience that can be applied to everyday life, instead of being left in the classroom.

1. **Make sense of problems and persevere in solving them.**

   The first Common Core mathematical practice standard is found in almost every math problem across the board. It means that students must understand the problem, figure out how to solve it, and then work until it is finished. Common Core standards encourage students to work with their current knowledge bank and apply the skills they already have while evaluating themselves in problem-solving. This standard is easily tested using problems with a tougher skill level than already mastered. While students work through more difficult problems, they focus on the process of solving the problem instead of just getting to the correct answer.

2. **Reason abstractly and quantitatively**

   When trying to problem solve, it is important that students understand there are multiple ways to break apart the problem in order to find the solution. Using symbols, pictures or other representations to describe the different sections of the problem will allow students to use context skills rather than standard algorithms.

3. **Construct viable arguments and critique the reasoning of others**

   This standard is aimed at creating a common mathematical language that can be used to discuss and explain math as well as support or object others’ work. Math vocabulary is easily integrated into daily lesson plans in order for students to be able to communicate effectively. “Talk moves” are important in developing and building communication skills and can include such simple tasks as restating a fellow classmate’s reasoning or even supporting their own reason for agreeing or disagreeing. Prompting students to participate further in class mathematical discussion will help build student communication skills.

4. **Model with mathematics**

   Math doesn’t end at the classroom door. Learning to model with mathematics means that students will use math skills to problem-solve real world situations. This can range from organizing different types of data to using math to help understand life connections. Using real world situations to show how math can be used in many different aspects of life helps math to be relevant outside of math class.
5. **Use appropriate tools strategically**

One of the Common Core’s biggest components is to provide students with the assets they need to navigate the real world. In order for students to learn what tools should be used in problem solving it is important to remember that no one will be guiding students through the real world – telling them which mathematics tool to use. By leaving the problem open ended, students can select which math tools to use and discuss what worked and what didn’t.

6. **Attend to precision**

Math, like other subjects, involves precision and exact answers. When speaking and problem-solving in math, exactness and attention to detail is important because a misstep or inaccurate answer in math can be translated to affect greater problem-solving in the real world. The importance in this step comes in the speaking demeanor of students to explain what is understood and what isn’t.

7. **Look for and make use of structure**

When students can identify different strategies for problem solving, they can use many different skills to determine the answer. Identifying similar patterns in mathematics can be used to solve problems that are out of their learning comfort zone. Repeated reasoning helps bring structure to more complex problems that might be able to be solved using multiple tools when the problem is broken apart into separate parts.

8. **Look for and express regularity in repeated reasoning**

In mathematics, it is easy to forget the big picture while working on the details of the problem. In order for students to understand how a problem can be applied to other problems, they should work on applying their mathematical reasoning to various situations and problems. If a student can solve one problem the way it was taught, it is important that they also can relay that problem-solving technique to other problems.
Identifying High-Quality Mathematics Tasks or Supplemental Resources

The following tool identifies characteristics that are consistently found in high quality tasks. The following rating or review tool should be used to help identify if a mathematical task or supplemental resource is of high quality. It is important to keep in mind that there is no perfect task. Every task can be improved. The tool can be applied to print resources as well as online resources.

### Identifying High-Quality Tasks

The purpose of the task is to teach or assess:

<table>
<thead>
<tr>
<th>Conceptual Understanding</th>
<th>Procedural skill and fluency</th>
<th>Application</th>
</tr>
</thead>
</table>

**Rating Scale:**

- 2 - Fully Meets the Characteristic
- 1 - Partially Meets the Characteristic
- 0 - Does Not Meet the Characteristic

<table>
<thead>
<tr>
<th>The mathematics task</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aligns to mathematics content standards I am teaching.</td>
<td></td>
</tr>
<tr>
<td>Encourages my students to use representations.</td>
<td></td>
</tr>
<tr>
<td>Provides my students with an opportunity for communicating their reasoning.</td>
<td></td>
</tr>
<tr>
<td>Has multiple entry points.</td>
<td></td>
</tr>
<tr>
<td>Allows for different strategies for finding solutions.</td>
<td></td>
</tr>
<tr>
<td>Makes connections between mathematical concepts, between concepts and procedures, or between concepts, procedures, and application.</td>
<td></td>
</tr>
<tr>
<td>Prompts cognitive effort.</td>
<td></td>
</tr>
<tr>
<td>Is problem-based, authentic, or interesting.</td>
<td></td>
</tr>
</tbody>
</table>

*(Retrieved from Mine the Gap for Mathematical Understanding By: John SanGiovanni 2017)*
## The task aligns to the mathematics standards I am teaching.
Tasks must be worthwhile and aligned to the skills and concepts in our curriculum.

<table>
<thead>
<tr>
<th>Tasks that fully meet this characteristic align directly to standards in my curriculum.</th>
<th>Tasks that partially meet this characteristic align to a standard in an adjacent grade level but are important and necessary.</th>
<th>Tasks that do not meet this characteristic do not connect with standards in my curriculum.</th>
</tr>
</thead>
</table>

## The task encourages my students to use representations.
Representations help students make sense of and communicate mathematical ideas.

<table>
<thead>
<tr>
<th>Tasks that fully meet this characteristic explicitly direct students to use representations.</th>
<th>Tasks that partially meet this characteristic imply or provide space for representations.</th>
<th>Tasks that do not meet this characteristic are clearly procedural with no reference or space for representations.</th>
</tr>
</thead>
</table>

## The task provides my students with an opportunity for communicating their reasoning.
Students can communicate their reasoning with models or pictures, numbers, and words.

<table>
<thead>
<tr>
<th>Tasks that fully meet this characteristic explicitly direct students to communicate their reasoning.</th>
<th>Tasks that partially meet this characteristic imply that students should communicate their reasoning.</th>
<th>Tasks that do not meet this characteristic do not require students to explain or justify their thinking.</th>
</tr>
</thead>
</table>

## The task has multiple entry points.
Students can approach a problem from various perspectives, using diverse strategies and/or representations.

<table>
<thead>
<tr>
<th>Tasks that fully meet this characteristic are open to many possible solution paths and representations.</th>
<th>Tasks that partially meet this characteristic can be approached in different ways but may provide an example or prompt to direct students to an approach.</th>
<th>Tasks that do not meet this characteristic have a specific solution path intended or directed.</th>
</tr>
</thead>
</table>

*(Retrieved from Mine the Gap for Mathematical Understanding By: John SanGiovanni 2017)*

Updated May 2018
Identifying High-Quality Mathematics Tasks or Supplemental Resources (pg. 3)

The task allows for different strategies for finding solutions.
Students can solve a problem in various ways.

| Tasks that fully meet this characteristic are open to any strategy regardless of the efficiency of the strategy. | Tasks that partially meet this characteristic can be approached in different ways but imply a specific strategy for students to use. | Tasks that do not meet this characteristic direct students to a specific solution path or calculation. |

The task makes connections between mathematical concepts.
Mathematics ideas are related. We can also connect them to representations, procedures, and applications.

| Tasks that fully meet this characteristic connect mathematical ideas or connect concepts/procedures/applications within a topic. | Tasks that partially meet this characteristic allow for connections but do not call for them directly. | Tasks that do not meet this characteristic make no connections. They focus on a single procedure or recall. |

Task prompts cognitive effort.
High-quality tasks should generate some amount of struggle. Students should have to make sense of the prompt, the problem, or the representation.

| Tasks that fully meet this characteristic offer no obvious solution path. Or tasks require concepts and procedures to be applied to new situations or contexts. | Tasks that partially meet this characteristic are problem based but indicate how they can be solved. | Tasks that do not meet this characteristic provide no cognitive resistance. Students are directed to do something exact or recall a skill or concept. |

Tasks are problem based, authentic, or interesting.
High-quality tasks are problem based. They can reflect real-world, authentic applications of mathematics. They should have interesting or novel prompts that grab students’ attention.

| Tasks that fully meet this characteristic are problem based and authentic or interesting. | Tasks that partially meet this characteristic are problem based. | Tasks that do not meet this characteristic are not problem based. |

(Retrieved from Mine the Gap for Mathematical Understanding By: John SanGiovanni 2017)

Updated May 2018
USD #457 Mathematics Framework

Instructional Components of Eureka Math

- **Fluency Practice**: (whole group)
  - promotes automaticity
  - students are engaged
  - high paced and energetic

- **Application Problem**: (whole group)
  - independent and/or collaborative
  - Kagan structures utilized
  - student discourse present-discovers, recognizes, and verbalizes connections
  - students understand and utilize RDW
  - evidence of movement from concrete to representational to abstract
  - students are given the opportunity to solve the problem without teacher guidance
- **Concept Development (6+ Classwork)**: (whole group)
  - evidence of movement from concrete to representation to abstract (CRA)
  - appropriate time is given to establish new learning
  - teacher checks for understanding and provides immediate feedback
  - student engagement structures may be used (ex: Kagan)
  - student discourse present
  - **Problem Set** is utilized in the concept development or as a result of the concept development

- **Debrief (6+ Closing)**: (whole group)
  - PINNACLE of the lesson, if you are short on time, PROTECT THE DEBRIEF
  - students articulate the focus of the lesson (metacognition)
  - students identify connections between parts of the lesson and/or previous taught concepts
  - teachers use rigorous questions to engage students in mathematical dialogue

- **Exit Ticket** (Independent)
  - students are accountable for the day’s learning
  - teacher uses tickets to inform instruction

- **Homework** (Independent)
  - reinforces already taught concepts
  - builds student confidence
  - checks for understanding and confirms independent proficiency

- **Centers (optional)** (small group/independent)
  - centers can be utilized after the completion of all lesson components
  - a balance must be present between conceptual vs procedural activities (Suggested 50/50 balance)
  - aligned to KCCRS and Eureka Math curriculum
  - can be a review of skills already taught in current or previous grade levels
## USD #457 Mathematics Resource Protocol

<table>
<thead>
<tr>
<th>Grade Levels</th>
<th>Core Resources</th>
<th>Approved Supplemental Resources</th>
<th>Intervention Programs (Tier 2 and 3)</th>
</tr>
</thead>
</table>
| Elementary (K-5th)| Eureka Math [https://greatminds.org](https://greatminds.org) | **Problem Solving:**  
  - Youcubed [https://www.youcubed.org](https://www.youcubed.org)  
  - Inside Mathematics [www.insidemathematics.org](http://www.insidemathematics.org)  
  - 3-Act Math Tasks K-6 [https://gfletchy.com/3-act-lessons/](https://gfletchy.com/3-act-lessons/)  
  - Nrich Enriching Mathematics [https://nrich.maths.org](https://nrich.maths.org)  
  - Problem Solving In All Seasons By: Kim Markworth, Jenni McCool, & Jennifer Kosiak  
  - Federal Way Public Schools Problem Solving Activities (Activities Aligned to Eureka Math Modules) [https://www.fwps.org/page/2060](https://www.fwps.org/page/2060)  
  - Estimation 180 [www.estimation180.com](http://www.estimation180.com)  
  - Scholastic Math magazine  
  - Mathematics Georgia Standards of Excellence [www.georgiastandards.org](http://www.georgiastandards.org)  
  - Number Worlds  
  - DreamBox  
  - iStation Math  
  - Eureka Math (pre-teaching, re-teaching)  
  - Zearn [http://www.zearn.org](http://www.zearn.org)  
  - ECAM diagnostic assessment and resources (K-2)  
  - Number Readiness diagnostic assessment (1st-5th) and resources  
  - Mathematics Georgia Standards of Excellence [www.georgia standards.org](http://www.georgia standards.org) |
|                   | **Fluency Support:**  
  - Math Fact Strategies Books (Digital Resource book)  
  - Greg Tang Math [www.gregtangmath.com](http://www.gregtangmath.com)  
  - Mastering Basic Facts Add/Subtraction and Multiplication/Division By: John San Giovanni  
  - Well Played By: Linda Dacey, Karen Gartland, & Jayne Bamford Lynch  
  - Elementary Number Talks (Online Resources) [https://elementarynumbertalks.wordpress.com](https://elementarynumbertalks.wordpress.com) | |
|                   | **Content Support:**  
  - Illustrative Mathematics [https://www.illustrativemathematics.org](https://www.illustrativemathematics.org)  
  - Illuminations [http://illuminations.nctm.org](http://illuminations.nctm.org)  
  - K-5 Math Teaching Resources [www.k-5mathteachingresources.com](http://www.k-5mathteachingresources.com)  
  - Mine the Gap for Mathematical Understanding By: John SanGiovanni  
  - Zearn [https://www.zearn.org](https://www.zearn.org)  
  - Eureka Math Bay Area Regional Consortium [https://embarc.online/](https://embarc.online/)  
  - Mathematics Georgia Standards of Excellence [www.georgiastandards.org](http://www.georgiastandards.org) | |
## USD #457 Mathematics Resource Protocol (pg.2)

<table>
<thead>
<tr>
<th>Middle (6th-8th)</th>
<th>Eureka Math</th>
<th>Problem Solving:</th>
<th>Do the Math</th>
</tr>
</thead>
</table>
|                   | https://greatminds.org | • Youcubed https://www.youcubed.org/  
• Inside Mathematics www.insidemathematics.org  
• 3-Act Math Tasks 6-  
HS: https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFejIm2xnhgwOVsWkTQEfqouLFN7NC6Z4/edit#gid=0  
• 3-Act Math Tasks K-6 https://gfletchy.com/3-act-lessons/  
• NRich Enriching Mathematics https://nrich.maths.org  
• Estimation 180 www.estimation180.com  
• Robert Kaplinsky Problem Based Lessons https://robertkaplinsky.com/lessons/ | Number Worlds  
DreamBox  
iStation Math  
Eureka Math (pre-teaching, re-teaching)  
Zearn http://www.zearn.org |

| Fluency Support: | | |
|------------------|----------------------------------| |
|                   | • Math Fact Strategies Books (Digital Resource book)  
• Building Conceptual Understanding and Fluency Through Games – North Carolina (Digital Resource book)  
• Greg Tang Math www.gregtangmath.com  
• Mastering Basic Facts Add/Subtraction and Multiplication/Division By: John San Giovanni  
• Well Played By: Linda Dacey, Karen Gartland, & Jayne Bamford Lynch | |

| Content Support: | | |
|------------------|----------------------------------| |
|                   | • Illustrative Mathematics https://www.illustrativemathematics.org/  
• ILLuminations http://illuminations.nctm.org/  
• K-5 Math Teaching Resources www.k-5mathteachingresources.com  
• Mine the Gap for Mathematical Understanding By: John San Giovanni  
• Zearn https://www.zearn.org  
• Eureka Math Bay Area Regional Consortium https://embarc.online/  
• KSDE Top Math Website Resources http://community.ksde.org/Default.aspx?tabid=6173 | |

| High (9th – 12th) | Agile Minds (Algebra I and Intensified Algebra) | Problem Solving: | |
|------------------|----------------------------------|------------------| |
|                   |                                   | • Youcubed https://www.youcubed.org/  
• Inside Mathematics www.insidemathematics.org  
• 3-Act Math Tasks 6-  
HS: https://docs.google.com/spreadsheets/d/1jXSt_CoDzyDFejIm2xnhgwOVsWkTQEfqouLFN7NC6Z4/edit#gid=0  
• 3-Act Math Tasks K-6 https://gfletchy.com/3-act-lessons/  
• NRich Enriching Mathematics https://nrich.maths.org  
• Estimation 180 www.estimation180.com  
• Robert Kaplinsky Problem Based Lessons https://robertkaplinsky.com/lessons/ | Do the Math  
Number Worlds  
DreamBox  
iStation Math  
Eureka Math (pre-teaching, re-teaching)  
Zearn http://www.zearn.org |

| Fluency Support: | | |
|------------------|----------------------------------| |
# USD #457 Mathematics Resource Protocol (pg. 3)

## Recommended Teacher Resources (K-12)

<table>
<thead>
<tr>
<th>Resource</th>
<th>Authors/Contributors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principles to Action</td>
<td>NCTM</td>
</tr>
<tr>
<td>Putting the Practices into Action</td>
<td>John SanGiovanni &amp; Susanne O’Connell</td>
</tr>
<tr>
<td>Teaching Student-Centered Mathematics (PreK-2), (3-5), and (6-8) (CCSS updated version)</td>
<td>John Van de Walle</td>
</tr>
<tr>
<td>5 Practices for Orchestrating Productive Mathematics Discussion</td>
<td>Mary K. Stein &amp; Margaret Schwan Smith</td>
</tr>
<tr>
<td>Classroom Discussions</td>
<td>Suzanne Chapin</td>
</tr>
<tr>
<td>Number Talks (K-5), and (6-8)</td>
<td>Sherry Parris</td>
</tr>
<tr>
<td>Taking Action: Implementing Effective Mathematics Teaching Practices</td>
<td>Various Authors</td>
</tr>
<tr>
<td>KATM Mathematics Flipbooks</td>
<td></td>
</tr>
<tr>
<td>Mathematical Mindsets</td>
<td>Jo Boaler</td>
</tr>
<tr>
<td>Mathematics Learning Progressions</td>
<td></td>
</tr>
<tr>
<td>KSDE Math Website</td>
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</table>

## Home Support Resources

<table>
<thead>
<tr>
<th>Resource</th>
<th>Website</th>
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</thead>
<tbody>
<tr>
<td>Freckle Education</td>
<td><a href="http://www.freckled.com">www.Freckled.com</a></td>
</tr>
<tr>
<td>Khan Academy</td>
<td><a href="http://www.khanacademy.com">www.Khanacademy.com</a></td>
</tr>
<tr>
<td>Prodigy Math Game</td>
<td><a href="http://www.prodigy.com">www.Prodigy.com</a></td>
</tr>
<tr>
<td>Sumdog</td>
<td><a href="http://www.sumdog.com">www.Sumdog.com</a></td>
</tr>
<tr>
<td>Eureka Math</td>
<td><a href="http://www.greatminds.org">www.greatminds.org</a></td>
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<td>Zearn</td>
<td><a href="https://www.zearn.org">https://www.zearn.org</a></td>
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